

REVIEW

Carl Wirtz' article from 1924 in *Astronomische Nachrichten* on the radial motions of spiral nebulae

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In the year 1924, a paper by Carl Wirtz appeared in "Astronomische Nachrichten", entitled "De Sitter's cosmology and the radial motion of spiral galaxies". This paper and its author remained largely unnoticed by the community, but it seems to be the first cosmological interpretation of the redshift of galaxies as a time dilation effect and the expansion of the Universe. Edwin Hubble knew Wirtz' publications quite well. The modern reader would find Wirtz' own understanding diffuse and contradictory in some aspects, but that reflected the early literature on nebulae, to which he himself made important contributions. The 100th anniversary provides a good opportunity to present an English transcription to the community, which can be found in the appendix. This anniversary also provokes to ask for the present status of cosmology which many authors see in a crisis. From an observational viewpoint it shall be illustrated that until today there is no consistent/convincing understanding of how the Universe evolved.

KEYWORDS:

cosmology: observations – history and philosophy of astronomy

1 | CARL WIRTZ AND THE EXPANSION OF THE UNIVERSE, THE BEGINNING OF MODERN OBSERVATIONAL COSMOLOGY

1.1 | Carl Wirtz is not prominent

The discovery of the expansion of the Universe marked the beginning of modern observational cosmology. How that happened was rather a process than a moment (e.g. Cervantes-Cota, Galindo-Uribarri, and Smoot 2023; Grøn 2018; van den Bergh 2011) and the prominent names are known to every astronomer and with Einstein even outside the astronomical community. Relativistic cosmology has been founded by Einstein himself and de Sitter, but de Sitter was also a key author for making Einstein's theory quickly known to the English speaking community (de Sitter, 1916a, 1916b) (de Sitter wrote German articles as well). The same can be said for Eddington (1923) and Weyl (1922).

Friedmann's fundamental papers on world models as general solutions of Einstein's field equations (Friedmann, 1922, 1924) were written in German and were more or less ignored by the community for about a decade (although Einstein knew them well) until Robertson (1933) presented his review of relativistic cosmology. Friedmann already introduced the beginning of the Universe and described pulsating Universes with a period of the order 10^{10} years.

On the observational side, the question who discovered the expansion normally leads to the names of Lemaitre and Hubble. However, in the recent review article of Cervantes-Cota et al. (2023), a paragraph is also devoted to Carl Wirtz, a name that is not known to every astronomer. The low prominence the work of Carl Wirtz enjoys in comparison with Lemaitre and Hubble, is inappropriate.

There are a few modern references to Wirtz in the English literature, most notably Duerbeck (1989), Seitter and Duerbeck (1999) and Duerbeck (2002), now also

Cervantes-Cota et al. (2023). There are a few more in German in amateur journals, we cite Priester and Schaaf (1987), Duerbeck and Seitter (1990a), Duerbeck and Seitter (1990b), and Appenzeller (2009).

In the year 1924, Wirtz published in "Astronomische Nachrichten" a paper (Wirtz, 1924) which has the characteristics to count as the starting point of modern observational cosmology (at the time of writing: 22 citations). We find its 100th anniversary a nice opportunity to present that short paper in an English translation to the community in the same journal. The translated article is presented in the Appendix.

Basic biographical dates: Carl Wirtz was born 1876 in Krefeld and died 1938 in Hamburg. He got his PhD 1899 at Bonn University, worked as assistant in Vienna until 1901, then moved to Strasbourg, where he got a professorship in 1909 and stayed there until 1919. Then he moved to Kiel as professor and was forced into early retirement 1937. In the following, the work of Carl Wirtz shall be briefly introduced in the context of cosmological quest in the early 20th century.

1.2 | Earlier work of Carl Wirtz in the context of cosmology from 1915 -1923

It was Schwarzschild who introduced the concept of curvature of space in cosmology (Schwarzschild, 1900; Stewart, Stewart, & Schwarzschild, 1998). He estimated the minimal curvature radius of the Universe to be approximately 10^8 astronomical units by consideration of the maximal inhomogeneity of the distribution of stars. When Einstein (1915) published his famous equation connecting gravity with the geometry of space-time, the Universe was still commonly understood as consisting mainly of a stellar system populating a large volume, as Schwarzschild did. The distinction between Galactic and extragalactic objects had no reason. The distances and nature of the spiral nebulae were unknown.

de Sitter (1917) published his world models as cosmological solutions to Einstein's equations. De Sitter's Universe (his model B) was finite with a certain curvature radius R , static and empty. Time delation with distance r resulted through the metric tensor element g_{44} which in the four-dimensional line element $ds^2 = \dots + \cos^2(r/R)c^2dt^2$ effectively gives a non-linear relation between redshift and distance. de Sitter himself tried to identify cosmological redshifts among various types of stars and only had the Andromeda nebula, NGC 1068, and NGC 4696 as extragalactic objects. But Andromeda is approaching and the other two galaxies show quite similar velocities about 1000 km/s, so the effect of increasing time dilation with distance which he hoped to see, was not visible.

At about the same time Wirtz (1918) wrote a paper on proper motions of 378 spiral nebulae. That was a topic that he had

addressed earlier in several contributions. He used the (spurious) proper motions to derive an apex of the solar motion quite similar to that from stellar streams. The mean proper motion for the spirals resulted to 0.027 "/year. However, this time he also compiled 15 radial velocities of spiral galaxies and noted the higher velocities with respect to the stellar system, but also that a common velocity and a vertex direction is not a good representation. The introduction of a constant K independent from the coordinates at the sphere led to a much better representation through the equation

$$v = V_x * \cos\alpha \cos\delta + V_y * \sin\alpha \cos\delta + V_z * \sin\delta + K \quad (1)$$

v being the velocity of an individual object at spherical coordinates α and δ , and V_x, V_y, V_z the velocity components of the (negative) apex motion of the observer.

He got the best fit of the above relation for $V_x = -145$ km/s, $V_y = -268$ km/s, $V_z = -762$ km/s (i.e. $V = -820$ km/s), and $K = +656$ km/s, and interpreted V as the radial correspondence to the proper motion. Then it is straightforward to derive a mean distance for the spiral nebulae. His result was 6.66 kpc and he concluded that the spiral nebulae are located outside the Milky Way system. He then tried to interpret the "strange" constant K . In his own words: *If we give this value a literal interpretation, then it means that the system of spiral nebulae drives apart with respect to the actual position of the solar system as the centre with a velocity of +656 km/s. The paper concludes: Also in the case of the nebulae one expects that we hold single threads of a mesh in hands whose complete pattern we still cannot disentangle.*

Four years later Wirtz published a paper entitled *On the statistics of the radial motions of spiral nebulae and globular clusters* (Wirtz, 1922), again in "Astronomische Nachrichten". Strikingly, proper motions of the nebulae that had accompanied his scientific career for a long time, were not mentioned at all. Apparently this chapter had been closed with the previous paper. Now Wirtz lists only radial velocities of 29 spiral nebulae and 10 globular clusters. For the spiral nebulae, he finds a somewhat higher K -term of +887 km/s, meaning that the entire system is moving away with this common radial velocity. He now considers the residuals to this common movements which he calls "errors" and understands them as the peculiar velocities of the spiral galaxies. For these velocities, he finds a slight dependence on Galactic latitude in the sense that the polar nebulae have higher velocities and on apparent magnitude, where the brightest nebulae are approaching and the fainter ones receding. This dependence also appears with the angular diameters.

Wirtz: *Of course, one finds the analog relation also with the diameter of the nebulae, where small diameters belong to*

a positive change of the distance, large diameters to a negative change. These statistical phenomena occur on top of the most striking and principal instance that can be described as if the system of spiral nebulae was drifting apart relative to our position... and the dependence on magnitude indicates that the nearest or the most massive spiral nebulae show a slower outward motion than the more distant or the less massive ones.

Here appears for the first time the notion of a radial velocity dependent on distance, but disguised as a second order effect that is still covered by the imagination of a system of spiral galaxies moving away with a constant velocity. In modern language, Wirtz had to deal with the fact that many of his galaxies are not in the Hubble-flow.

Wirtz' main observational work in Strasbourg was the photometry of nebulae, including globular clusters. This resulted in a catalogue of surface brightnesses, total brightnesses, and angular diameters for 566 objects (Wirtz, 1923). Presumably, Wirtz saw that apparent magnitudes and angular diameters are not well correlated and therefore did not use magnitudes as distance indicators.

1.3 | The role of Friedmann

Two publications of the mathematician, meteorologist, geophysicist, pilot and adventurer dominated the cosmological field for (at least!) 100 years. In a static Universe the old mystification of circular orbits may have survived. Planetary orbits must(!) be circular, elliptical orbits are ugly. And the Universe cannot be variable! In this analogy, Friedmann's role is the closest to Kepler's. In an expanding Friedmann universe, time dilation results naturally as proportional to $1+z$ and a linear velocity-distance relation is necessary to maintain homogeneity.

In 1922 Friedmann (1922) already introduced "non-stationary" world models (in German). There are no indications that Wirtz was aware of Friedmann's work, but even Einstein needed some time to see that Friedmann's solutions were indeed correct. The most famous physicist of the epoch gives in a brief note his blessings to the concept of an expanding Universe and is ignored himself (Einstein, 1923)! Perhaps the German language again was an obstacle for the international community. Also Friedmann's last paper (Friedmann, 1924) did not express interest in observational evidence for his models.

1.4 | The work of Silberstein, Oepik, Lundmark and Strömberg

In the same year 1924 appeared the work of Silberstein (1924). Silberstein understood de Sitter's time dilation as a relativistic Doppler effect and derived a relation between redshift and

curvature radius of the Universe. To determine this radius R , he used the radial velocities of 7 globular clusters and got $R = 1.1 \times 10^7$ pc. Silberstein (1929) still argued strongly for a value of 1.67×10^6 pc, meaning that in his de Sitter model, the largest possible distance in the Universe would be 1.3×10^6 pc, contradicting strongly the by then established extragalactic distances.

The extragalactic nature of the Andromeda nebula was by 1922 quite clear. Oepik (1922), by using the rotational velocity and applying an early version of the Tully-Fisher relation, got a distance of 500 000 pc, three years before E. P. Hubble (1925) published in a much more famous paper the Cepheid distance of NGC 6822 of 214 000 pc, in Hubble's words *the first object definitely assigned to a region outside the galactic system*.

However, it is Lundmark (1924a) who deserved the credit to have shown the first "Hubble-diagram", although a velocity-distance relation is not recognisable in his graph. As for Silberstein, his main objective was to measure the curvature radius of a de Sitter Universe, and not the nature of spiral galaxies. For that he needed absolute distances for the spiral nebulae. He used the Andromeda nebulae as the reference distance of 200 000 pc which he got from comparing Galactic novae with novae in Andromeda. He explicitly did not exclude a distance of 500 000 pc, as favoured by Oepik (1922).

Lundmark adopted Silberstein's formula and interpretation, but with a much enlarged data set, ranging from nearby stars over Cepheids and novae to the sample of spiral galaxies which had been used also by Wirtz (he apparently got the spiral galaxy data personally from Slipher, Lundmark 1924b). Lundmark determined individual distances for his spiral galaxies through the comparison with Andromeda regarding brightnesses and angular diameters. Without an established distance scale, the individual errors are of course large, particularly the three most distant galaxies (on Lundmark's scale) NGC 278, NGC 1700, and NGC 2681 show moderate velocities around 700 km/s. And many nebulae are, in modern terms, not in the Hubble flow, unfortunately also Andromeda which is the reference. Lundmark also gives "mean distances" for his sample by the comparison of the K -term with (spurious) proper motions. Although he labels the proper motion values as "illusory" and "only of use for giving a upper limit to the motion", the mere fact that there is a common proper motion of spiral nebulae, clashed with the concept of expansion.

A third author who used Slipher's radial velocities in a cosmological context was Gustaf Strömberg (Stromberg, 1925). He performed his analysis without the knowledge of Lundmark's results, but expressed in the published paper his satisfaction about the agreement with the main result, namely that there is no relation of redshift with distance. Strömberg calculated the distances for spiral nebulae as $10^{0.2m}$ where the m are apparent magnitudes as given by Wirtz (1923)! He only gives

correlation coefficients. The strongest correlation has been found with the angular distance from the sun's apex. As one sees already from Lundmarks sample, the individual distances based on magnitudes are simply too uncertain to uncover any velocity-distance relation. One may find some irony in the fact that Wirtz' own measurements are used against his findings.

Strömberg moreover understood the redshifts as caused by de Sitter's time dilation not as indicating a real motion, which he considers to be "fictitious".

1.5 | Wirtz' influence on Hubble

To what degree Hubble was influenced by the reading of Wirtz' publications will remain elusive in detail (see Way 2013 for more comments on Hubble's work), but there is no doubt that Hubble knew Wirtz' 1924 paper (and others) quite well. The German language apparently was no obstacle to him. He cited regularly German papers, but firstly not Wirtz, particularly not in E. Hubble (1929) nor in E. Hubble and Humason (1931). However, Wirtz occupies a quite prominent place in Hubble's book "The realm of the nebulae" from 1936. In the chapter "The velocity-distance relation" Hubble describes the content of Wirtz (1918) and Wirtz (1922) in some detail (for the latter paper, he cites the year as 1921, not 1922). But he praises Wirtz mainly as the discoverer of the "K-term", the mean residual of radial velocities of the nebulae after correction for solar motion. Hubble calls Wirtz even "the leader in the field". He qualifies the results of the 1924 paper as "suggestive rather than definitive". In conjunction with the discussion of the work of Lundmark (1924b) and Strömberg (1925), Hubble concluded that by 1925, "the data did not establish a relation" (between distance and radial velocity) and so repeated Lundmark's and Strömberg's statements, probably without looking further into the details of their distance determinations. Nevertheless, Hubble adds a footnote "Wirtz later published a stimulating popular presentation of the investigation and the implications of the results (*Scientia*, 38, 303, 1925) in which he assumes that de Sitter's prediction has been verified."

The 100 years after Wirtz' paper have mainly seen an acceptance of Friedmann's understanding of the expansion of the Universe, culminating in the emergence of the "standard cosmology" with Dark Matter and Dark Energy as its pillars, simultaneously its threats.

2 | DO WE REALLY UNDERSTAND THE COSMOS?

I shall give a brief account of literature with the emphasis on those works that represent the main problematic issues in modern cosmology, and that do not appear in otherwise

exhaustive reviews of the history of the Hubble constant (e.g. Cervantes-Cota et al. 2023). The breath-taking progress in both observations and theory let any review to be quickly outdated (e.g. López-Corredoira 2017, Amendola et al. 2013). This section shall demonstrate that the full understanding of galaxy redshifts is a process that until today is still going on.

2.1 | Standard cosmology and challenges

The following sampling of literature highlights the search for observable expansion signatures and shall illustrate that the "standard cosmology" faces serious challenges. The section title plagiarises Padmanabhan (2017) who analyses profoundly cosmological fundamentals from the viewpoint of a theoretical physicist. As for the phenomenon of expansion, he asks "How come a cosmological arrow of time emerges from equations of motion which are time-reversal invariant?" and suggests to search the answer in a quantum description of spacetime. For humble astronomers, the directly observable (at least principally) properties of expansion, most notably the value of the Hubble constant and the deceleration parameter, formed the core of the cosmological endeavour for a long time (A. Sandage, 1998, 1999; A. R. Sandage, 1970).

The spectacular successes in understanding the Cosmic Microwave Background, the distance scale and the large scale structure joined to formulate a "standard cosmology" where the total energy density of a Friedmann Universe is the sum of a baryonic part, Dark Matter, and Dark Energy (e.g. Planck Collaboration et al. 2015). The current price to be paid for successful fits of observed data of various kinds to a Friedmann cosmological scheme is that the major part of the energy density remains unexplained. However, dark matter and dark energy may not exist at all (see sections 2.3 and 2.4). There is hope that data from the James Webb Space Telescope (JWST) and other big telescopes to come will lead to a better understanding in the near future (e.g. Lovyagin, Raikov, Yershov, and Lovyagin 2022).

2.2 | Expansion signatures: time dilation and angular distances

Time dilation proportional to $(1+z)$ as a signature of expansion has been first observed in the broadening of SNIa lightcurves (Goldhaber et al., 1996; Leibundgut et al., 1996), which later was confirmed by a larger set of SNIa of higher redshifts (Blondin et al., 2008).

Quasar variability was another high-redshift phenomenon for which time dilation could be expected. For a long time irregular variability covered any potential signal and Hawkins (2010) firmly concluded that there is no time dilation. This

is contradicted by Lewis and Brewer (2023) who used long-term photometry of a large sample of quasars and indeed found cosmological time dilation.

Another signature of expansion is the growth of angular sizes for high-redshift objects. Because nearby objects become smaller with increasing distances, there must be a maximum of angular distances in an expanding universe. The lack of convincing rods has long impeded a proof of the reality of this maximum (Melia & Yennapureddy, 2018), who also provide a history of this relation. Lovyagin et al. (2022), collecting galaxy sizes recently measured with JWST (cautiously) conclude that the expanding-Universe model fails and that the galaxy ages might be much larger than expected for a Λ CDM Universe. This might be indicative of an early "loitering" phase of the Universe, when expansion almost stalled, which simultaneously boosted structure formation (Sahni, Feldman, & Stebbins, 1992).

2.3 | Accelerated expansion and backreaction

The apparent acceleration of the expansion had been worthy a Nobel price (Perlmutter et al., 1999; Riess et al., 1998; Schmidt et al., 1998) (together at the time of writing: 32038 citations). But see also Yoshii and Peterson (1995) who, by counting galaxies, probed volumes instead of distances. Being less explicit, they leave it to the reader, who is familiar with Friedmann models, to see an accelerated expansion in their Fig.2 (at the time of writing: 41 citations), and accordingly the existence of a cosmological constant or "Dark Energy" (e.g. Planck Collaboration et al. 2015). This concept has been criticised by authors who pointed out the non-linearity of the Einstein-equation and that a mean density, simply averaged over large volumes as it appears in the Friedmann-equation, is not an adequate description in a general relativistic framework. In their view, the cosmological constant emerges from a comparison of "apples with oranges", from comparing a Universe without structure (the CMB) with a Universe with structure (today's Universe). The effect of structure formation on the dynamical fate of the Universe has been called "backreaction" (Buchert, 2008; Buchert & Räsänen, 2012; Buchert, van Elst, & Heinesen, 2023; Wiltshire, 2007).

The apparent tension between a local H_0 and a "CMB- H_0 " provoked an intense discussion (Kroupa et al., 2023; Kumar Aluri et al., 2023). Again, it may well be traced back to the tension between a Universe with structure and a Universe without structure, whether inhomogeneities are global (Heinesen & Buchert, 2020) or local (Mazurenko, Banik, Kroupa, & Haslbauer, 2024).

2.4 | Dark Matter and MOND

The other main ingredient of Standard Cosmology is Dark Matter (from the huge literature we cite Bertone and Hooper 2018). However, all searches for particle dark matter in terrestrial detectors have been futile so far. Alternative concepts like Modified Newtonian Dynamics (Bekenstein and Milgrom 1984; Famaey and McGaugh 2012; Kroupa 2012, 2015; Kroupa, Pawłowski, and Milgrom 2012; Milgrom 1983a, 1983b, 1983c, 2023a) gained weight over the last decades. With today's knowledge of galaxy dynamics, the existence of MONDian phenomenology among galaxies is now well confirmed (e.g. Lelli, McGaugh, Schombert, and Pawłowski 2017; Richtler, Salinas, Lane, and Hilker 2024). Weak lensing data show the constancy of circular rotation curves of isolated galaxies out to hundreds of kpc, beyond the expected virial radii of dark matter halos (Brouwer et al., 2021; Mistele, McGaugh, Lelli, Schombert, & Li, 2024).

The long-standing allegation that MOND misses a theoretical basis, now appears unfounded. MOND can be understood as modified inertia rather than modified gravity (Milgrom, 2022, 2023b). Bekenstein (2004) was the first to present a relativistic theory with MOND as the non-relativistic limit, but that turned out to be incompatible with the equality of the speed of light and gravitational waves (Abbott et al., 2017). A new theory (Skordis & Złośnik, 2021; Skordis & Zlosnik, 2022) respects this important observation and reproduces the CMB structure.

One can also pave a direct way from the thermodynamics of horizons (Jacobson, 1995), describing gravity as emerging from microscopic degrees of freedom (Padmanabhan, 2015), to a MOND-like gravity as worked out by Hossenfelder (2017) and Hossenfelder and Mistele (2018).

MOND also may be dominant for early structure formation (Sanders, 1998; Wittenburg, Kroupa, Banik, Candlish, & Samaras, 2023), so backreaction effects are expected to be even stronger than in a standard Friedmann evolution. The influence MOND or its underlying theory on cosmology is only at the verge of being explored. It might dramatically change the current picture.

3 | FINAL REMARKS: TODAY'S RELEVANCE OF CARL WIRTZ

The few numbers in the table in Wirtz' paper may be seen as the first manifestation of one of the greatest discoveries in the history of physics. What the understanding of the contemporary authors was, we can only guess. But what can be shown is that it needed decades for the astronomical community to find a proper cosmological understanding of galaxy redshifts. It is very illustrative to scan through Appendix B of

Davis and Lineweaver (2004) and see what famous names have not been free of misconceptions.

Following again Padmanabhan (2017), modern cosmology is characterised by a disproportion between *knowledge* that increased tremendously during the last decades, and *understanding* that still uses the framework given by Friedmann. Wirtz and his fellow astronomers stand for the beginning of knowledge. Reading their papers today does not contribute to the understanding of cosmological expansion, but might contribute to the understanding of the emergence of modern cosmology.

4 | ACKNOWLEDGEMENTS

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APPENDIX A: CARL WIRTZ: DE SITTER'S COSMOLOGY AND THE RADIAL MOTIONS OF SPIRAL GALAXIES

The translation German - English is as literal as is justifiable. The unit km should be read as km/s.

Astronomische Nachrichten 1924, 222, 21W

The world of de Sitter is a spherical space-time world, a four-dimensional continuum of space and imaginary time that forms the surface of a sphere in five dimensions. De Sitter's world is devoid of mass, all mass swept away to a zone that is not accessible for observations, like a "mass-horizon" or a ring of peripheral matter, necessary to keep the emptiness within. The two limits that embrace the real world, are Einsteins world with a maximum of matter and de Sitter's world. W. de Sitter, *MN* 78.3, 1918; A.S. Eddington, *Math. Theory of Relativity*, Cambridge 1923, Chap III,V; compare also the illustrative graphical representation of de Sitter's world at H. Weyl, *Naturwiss.* 12 (1924), p.201.

In de Sitter's world, system B, occurs a phenomenon accessible to observations. The natural processes are slowing down with increasing

distance from the origin of coordinates, so do the natural clocks. In particular, atomic oscillations slow down. This decrease of the oscillatory periods can be observed: the spectral lines that come from distant light sources that are at rest in the used coordinate system, must be redshifted. A particle existing in empty space is pushed away from the origin with an acceleration that increases with distance. In de Sitter's world therefore two reasons for the redshift of spectral lines do exist: firstly the general dispersal of matter towards a matter horizon and secondly the spectral shift towards red which for distant objects occurs due to the slowing down of time, even if they are at rest with respect to the origin of coordinates.

Because of the symmetry of de Sitter's world formulae one can choose an arbitrary point as origin; meaning that there is no difference regarding the observed phenomena between the origin and the horizon. Any point can be origin, through every point of de Sitter's universe can pass the mass horizon, the unreachable periphery of space.

The conclusion for the stagnancy of time is rejected by de Sitter with the remark that all paradoxes of this world apply only to events that occur before or after eternity.

If one objects that de Sitter's world would not be static once matter has been brought in, then this property rather speaks in favour of its physical reality than against it.

Is it possible to observe the slowing down of natural processes with increasing distance?

de Sitter pointed already at the confirmed strong redshifts in the spectra of three spiral nebulae which, calculated as a Doppler effect, result in an average velocity of +600 km/s.

De Sitter's theory not only provides the mere fact of redshift, but, as mentioned, also the increase of this redshift with increasing distance and this for two reasons. This must appear in the observation in the manner that the Doppler radial motions of the spiral nebulae assume higher and higher positive values. The distance of the spiral nebulae are unknown; however, one might use the apparent diameter as a measure of the distance under the assumption of the same linear diameter on the average. In de Sitter's cosmology the radial velocities should increase with decreasing apparent diameter.

To prove these expectations, a sufficiently large sample of velocities is available that contains objects with large and small apparent

diameter. In its majority one owes these measurements, which are as difficult as valuable, to the work of V.M. Slipher, Lowell observatory. His values of radial motions are collected in a list that stems from February 1921 and which moreover contains some still unpublished data; it can be found at Eddington l.c. p.162. In addition, another source provides the radial motion for NGC 2681 of +700 km/s, so that velocities exist for 42 spiral nebulae. The photographic apparent diameters have been partly borrowed from H.D. Curtis, Lick Public. 13, P.I (1918), and partly from F.G. Pease, Mt. Wilson Contr. Nr.132 (1917), Nr. 186 (1920).

The following results of some little calculations are based on the nebulae major axes in arcmin, for obvious reasons not linearly, but in log Dm. After rebinning in suitable groups of the two arguments log Dm and v, one gets the table

TABLE A1 Radial velocities vs. angular diameter and vice versa

argum. log Dm			argum. v		
log Dm	v	n	v	log Dm	n
0.24	+827	9	-183	1.11	5
0.43	+656	7	+255	0.76	8
0.66	+512	8	+475	0.96	6
0.88	+555	10	+630	0.52	8
1.07	+334	5	+809	0.48	7
1.71	-20	3	+1160	0.60	8

One clearly recognises a relation in the sense that the positive radial motion decreases with increasing apparent diameter. Less strikingly, but still distinctively, one sees that with increasing velocity, the apparent diameter decreases. The degree of correlation of the two parameters, v and log Dm, is expressed by the correlation coefficient that has been determined from 42 pairs

$$r = -0.455 \pm 0.122, n = 42$$

Mean values: $v=+574$; $\log Dm=0.71$ (geom. mean=5'.1) with the linear regression lines:

$$v(km) = 914 - 479 \times \log Dm$$

and

$$\log Dm = 0.96 - 0.000432 \times v$$

There remains no doubt that the positive radial motion of spiral nebulae increases considerably with increasing distance. In reality, the mutual conditionality between v and log Dm may be stronger than the formal value of r indicates. In a plot of the individual values v against log Dm, the resulting distribution has a triangular or V-shape from which one reads off the following facts: among the small nebulae appear lowest and highest v, the apparently large nebulae have the lowest v, among nebulae with low v's, one finds small and large objects, large nebulae with high v do not exist.

Thus it can be concluded that the scatter of the linear nebulae dimensions fills the triangular area of the graph in the way that among the near nebulae with low v absolute small and large objects exist and are visible, while in the deep Universe only the absolute largest nebulae with high v are accessible for determination of their radial motions. Then the relation between log Dm and v would be best represented by the hypotenuse of the triangle, that limits the data points with the consideration that the giants among the nebulae have on the average the same linear dimensions at all distances. For these biggest nebulae the resulting formulae for the graphic representation is parenthetically

$$v(km) = 2200 - 1200 \times \log Dm$$

The coefficient 1200 expresses that a ten-fold increase of the distance, the redshift as a pure Doppler effect increases by 1200 km. This rule-of-thumb has an advantage: it does not clash with the speed of light, because the true and apparent radial velocities of distant matter that follow from the properties of de Sitter's space increase only slowly with distance. A redshift that would mean light velocity as a Doppler effect would only be reached in distances that lie beyond those nowadays believed to be the distances of heavenly bodies - about 10^{200} parsec, a value that exceeds all estimations for the radius of curved space.

For 32 of 42 nebulae with radial velocities, one can find photometric surface brightnesses in Lund Meddel.(2), 29, (1923), and it deserves mentioning in this context that the new larger data set gives the same correlation between radial velocity and surface brightness, as in LM, p.31; a correlation coefficient $r = +0.02 \pm 0.18$, i.e. there is no relation.

However, the positive sign indicates that radial motions increase with decreasing surface brightness. This sense one finds in all correlations that have been searched for where the distance of these entities is somehow involved. Regarding the absolute values at the borderline of reality, but regarding the sign always indicating a (very tiny) absorption in the depths of the universe to which one pushes the nebulae. Compare AN 222.33 (1924).

The relation between the radial motions of the nebulae and their distances has been recognised in another way already in AN 215, p. 352 (1922). It was shown that the intrinsic radial motions of the nebulae (with their signs) increase with increasing magnitudes of their total brightnesses. This relation is characterised by a correlation $r = +0.21$ from 19 objects, while here from 42 objects the correlation coefficient radial motion vs. $\log(\text{app. Dm})$ resulted as $r = -0.46$. A phenomenon that can be described in the above manner by the properties of de Sitter's world.

REFERENCES

- Abbott, B. P., Abbott, R., Abbott, T. D. et al. (2017, October), *ApJ*, 848(2), L13. doi:
- Amendola, L., Appleby, S., Bacon, D. et al. (2013, September), *Living Reviews in Relativity*, 16(1), 6. doi:
- Appenzeller, I. (2009, November), *Sterne und Weltraum*, 48, 44-52.
- Bekenstein, J. (2004, October), *Phys. Rev. D*, 70(8), 083509. doi:
- Bekenstein, J., & Milgrom, M. (1984, November), *ApJ*, 286, 7-14. doi:
- Bertone, G., & Hooper, D. (2018, October), *Reviews of Modern Physics*, 90(4), 045002. doi:
- Blondin, S., Davis, T. M., Krisciunas, K. et al. (2008, August), *ApJ*, 682(2), 724-736. doi:
- Brouwer, M. M., Oman, K. A., Valentijn, E. A. et al. (2021, June), *A&A*, 650, A113. doi:
- Buchert, T. (2008, February), *General Relativity and Gravitation*, 40(2-3), 467-527. doi:
- Buchert, T., & Räsänen, S. (2012, November), *Annual Review of Nuclear and Particle Science*, 62(1), 57-79. doi:
- Buchert, T., van Elst, H., & Heinesen, A. (2023, January), *General Relativity and Gravitation*, 55(1), 7. doi:
- Cervantes-Cota, J. L., Galindo-Uribarri, S., & Smoot, G. F. (2023, November), *Universe*, 9(12), 501. doi:
- Davis, T. M., & Lineweaver, C. H. (2004, January), *PASA*, 21(1), 97-109. doi:
- de Sitter, W. (1916a, June), *MNRAS*, 76, 699-728. doi:
- de Sitter, W. (1916b, December), *MNRAS*, 77, 155-184. doi:
- de Sitter, W. (1917, November), *MNRAS*, 78, 3-28. doi:
- Duerbeck, H. W. (1989), Carl Wirtz — An early observational cosmologist. In P. Flin & H. W. Duerbeck (Eds.), *Morphological Cosmology* Vol. 332, p. 405. doi:
- Duerbeck, H. W. (2002, July), *Astronomische Nachrichten*, 323(6), 534-537. doi:
- Duerbeck, H. W., & Seitter, W. C. (1990a, January), *Die Sterne*, 66(3), 131-139.
- Duerbeck, H. W., & Seitter, W. C. (1990b, January), *Die Sterne*, 66(3), 131-139.
- Eddington, A. S. 1923, The mathematical theory of relativity.
- Einstein, A. (1915, January), *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 844-847.
- Einstein, A. (1923, December), *Zeitschrift für Physik*, 16(1), 228-228. doi:
- Famaey, B., & McGaugh, S. S. (2012, December), *Living Reviews in Relativity*, 15(1), 10. doi:
- Friedmann, A. (1922, January), *Zeitschrift für Physik*, 10, 377-386. doi:
- Friedmann, A. (1924, December), *Zeitschrift für Physik*, 21(1), 326-332. doi:
- Goldhaber, G., Boyle, B., Bunclark, P. et al. (1996, November), *Nuclear Physics B Proceedings Supplements*, Vol. 51, 51, 123-127. doi:
- Grøn, Ø. (2018, December), *Galaxies*, 6(4), 132. doi:
- Hawkins, M. R. S. (2010, July), *MNRAS*, 405(3), 1940-1946. doi:
- Heinesen, A., & Buchert, T. (2020, August), *Classical and Quantum Gravity*, 37(16), 164001. doi:
- Hossenfelder, S. (2017, June), *Phys. Rev. D*, 95(12), 124018. doi:
- Hossenfelder, S., & Mistele, T. (2018, January), *International Journal of Modern Physics D*, 27(14), 1847010. doi:
- Hubble, E. (1929, March), *Proceedings of the National Academy of Science*, 15(3), 168-173. doi:
- Hubble, E., & Humason, M. L. (1931, July), *ApJ*, 74, 43. doi:
- Hubble, E. P. (1925, December), *ApJ*, 62, 409-433. doi:
- Jacobson, T. (1995, August), *Phys. Rev. Lett.*, 75(7), 1260-1263. doi:
- Kroupa, P. (2012, June), *PASA*, 29(4), 395-433. doi:
- Kroupa, P. (2015, February), *Canadian Journal of Physics*, 93(2), 169-202. doi:
- Kroupa, P., Gjergo, E., Asencio, E. et al. (2023, September), *arXiv e-prints*, arXiv:2309.11552. doi:
- Kroupa, P., Pawłowski, M., & Milgrom, M. (2012, December), *International Journal of Modern Physics D*, 21(14), 1230003. doi:
- Kumar Aluri, P., Cea, P., Chingangbam, P. et al. (2023, May), *Classical and Quantum Gravity*, 40(9), 094001. doi:
- Leibundgut, B., Schommer, R., Phillips, M. et al. (1996, July), *ApJ*, 466, L21. doi:
- Lelli, F., McGaugh, S. S., Schombert, J. M., & Pawłowski, M. S. (2017, February), *ApJ*, 836(2), 152. doi:
- Lewis, G. F., & Brewer, B. J. (2023, October), *Nature Astronomy*, 7, 1265-1269. doi:
- López-Corredoira, M. (2017, June), *Foundations of Physics*, 47(6), 711-768. doi:
- Lovayagin, N., Raikov, A., Yershov, V., & Lovayagin, Y. (2022, December), *Galaxies*, 10(6), 108. doi:
- Lundmark, K. (1924a, September), *The Observatory*, 47, 279-281.
- Lundmark, K. (1924b, June), *MNRAS*, 84, 747-770. doi:
- Mazurenko, S., Banik, I., Kroupa, P., & Haslbauer, M. (2024, January), *MNRAS*, 527(3), 4388-4396. doi:
- Melia, F., & Yennapureddy, M. K. (2018, October), *MNRAS*, 480(2), 2144-2152. doi:
- Milgrom, M. (1983a, July), *ApJ*, 270, 365-370. doi:
- Milgrom, M. (1983b, July), *ApJ*, 270, 371-383. doi:
- Milgrom, M. (1983c, July), *ApJ*, 270, 384-389. doi:
- Milgrom, M. (2022, September), *Phys. Rev. D*, 106(6), 064060. doi:
- Milgrom, M. (2023a, October), *Phys. Rev. D*, 108(8), 084005. doi:
- Milgrom, M. (2023b, October), *arXiv e-prints*, arXiv:2310.14334. doi:

- Mistele, T., McGaugh, S., Lelli, F., Schombert, J., & Li, P. (2024, July), *ApJ*, 969(1), L3. doi:
- Oepik, E. (1922, June), *ApJ*, 55, 406-410. doi:
- Padmanabhan, T. (2015, February), *Modern Physics Letters A*, 30, 1540007. doi:
- Padmanabhan, T. (2017, March), *Comptes Rendus Physique*, 18(3-4), 275-291. doi:
- Perlmutter, S., Aldering, G., Goldhaber, G. et al. (1999, June), *ApJ*, 517(2), 565-586. doi:
- Planck Collaboration, Ade, P. A. R., Aghanim, N. et al. (2015, February), *ArXiv e-prints*.
- Priester, W., & Schaaf, R. (1987, January), *Sterne und Weltraum*, 26, 376-377.
- Richtler, T., Salinas, R., Lane, R., & Hilker, M. (2024, January), *Astronomische Nachrichten*, 345(1), e20230081. doi:
- Riess, A. G., Filippenko, A. V., Challis, P. et al. (1998, September), *AJ*, 116(3), 1009-1038. doi:
- Robertson, H. P. (1933, January), *Reviews of Modern Physics*, 5(1), 62-90. doi:
- Sahni, V., Feldman, H., & Stebbins, A. (1992, January), *ApJ*, 385, 1. doi:
- Sandage, A. (1998, October), Beginnings of Observational Cosmology in Hubble's Time: Historical Overview. In M. Livio, S. M. Fall, & P. Madau (Eds.), *The Hubble Deep Field* p. 1.
- Sandage, A. (1999, November), *ApJ*, 525C, 252.
- Sandage, A. R. (1970, January), *Physics Today*, 23(2), 34-41. doi:
- Sanders, R. H. (1998, June), *MNRAS*, 296(4), 1009-1018. doi:
- Schmidt, B. P., Suntzeff, N. B., Phillips, M. M. et al. (1998, November), *ApJ*, 507(1), 46-63. doi:
- Schwarzschild, K. (1900, January), *Vierteljahrsschrift der Ast. Ges.*, 35, 337.
- Seitter, W. C., & Duerbeck, H. W. (1999, January), Carl Wilhelm Wirtz - Pioneer in Cosmic Dimensions. In D. Egret & A. Heck (Eds.), *Harmonizing Cosmic Distance Scales in a Post-HIPPARCOS Era* Vol. 167, p. 237-242.
- Silberstein, L. (1924, March), *Nature*, 113(2836), 350-351. doi:
- Silberstein, L. (1929, August), *Nature*, 124(3118), 179. doi:
- Skordis, C., & Złóśnik, T. (2021, October), *Phys. Rev. Lett.*, 127(16), 161302. doi:
- Skordis, C., & Złosnik, T. (2022, November), *Phys. Rev. D*, 106(10), 104041. doi:
- Stewart, J. M., Stewart, M. E., & Schwarzschild, K. (1998, September), *Classical and Quantum Gravity*, 15(9), 2539-2544. doi:
- Stromberg, G. (1925, June), *ApJ*, 61, 353-362. doi:
- Strömberg, G. (1925, June), *ApJ*, 61, 353-362. doi:
- van den Bergh, S. (2011, October), *JRASC*, 105(5), 197. doi:
- Way, M. J. (2013, April), Dismantling Hubble's Legacy? In M. J. Way & D. Hunter (Eds.), *Origins of the Expanding Universe: 1912-1932* Vol. 471, p. 97. doi:
- Weyl, H. 1922, Space - Time - Matter.
- Wiltshire, D. L. (2007, December), *Phys. Rev. Lett.*, 99(25), 251101. doi:
- Wirtz, C. (1918, March), *Astronomische Nachrichten*, 206(13), 109. doi:
- Wirtz, C. (1922, April), *Astronomische Nachrichten*, 215, 349. doi:
- Wirtz, C. (1923, January), *Meddelanden fran Lunds Astronomiska Observatorium Serie II*, 29, 5-63.
- Wirtz, C. (1924, July), *Astronomische Nachrichten*, 222(2), 21. doi:
- Wittenburg, N., Kroupa, P., Banik, I., Candlish, G., & Samaras, N. (2023, July), *MNRAS*, 523(1), 453-473. doi:
- Yoshii, Y., & Peterson, B. A. (1995, May), *ApJ*, 444, 15. doi:

